DP Physics HL

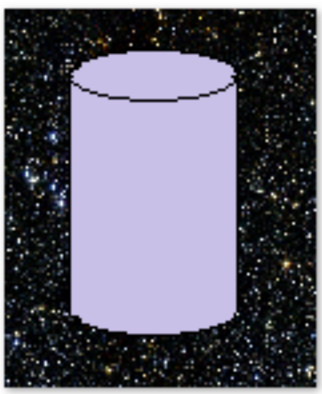
Static Fluids

****All simulations and videos required for this package can be found on my website, here:**

http://ismackinsey.weebly.com/fluids-hl.html

Fluids are substances that can flow, so this means liquids or gases. Static fluids refer to those that are not flowing.

Consider a cylinder containing a liquid:



- Does this fluid exert pressure on the sides of the container?

  Well, you know that if this were a water tank, it could potentially spring a leak at any loose rivets anywhere on the container, so the answer must be yes.

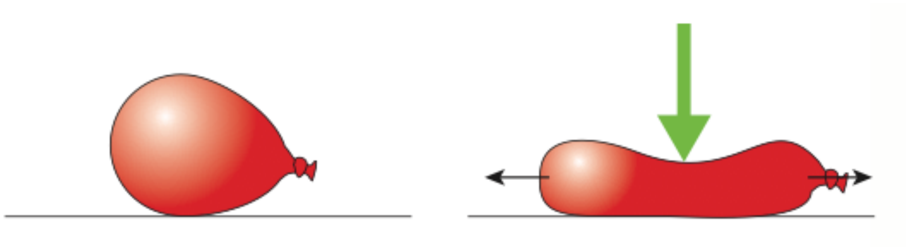
If the top of the cylinder were replaced by a piston, and a force were applied:

- Would the volume change?
- No, it can't because a liquid can't be compressed.

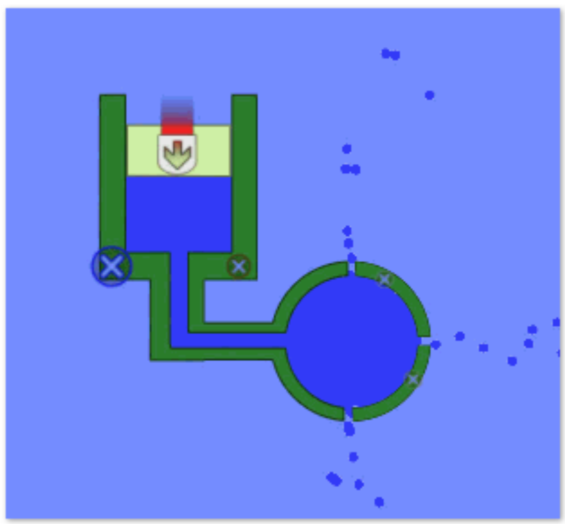
- What pressure would be exerted?
  There must be pressure on all sides of the cylinder. If the pressure on the piston is increase, the pressure throughout the liquid increases. This is **Pascal's Principle.**

Pascal's Principle: The pressure applied to a confined fluid increases the pressure throughout the fluid.

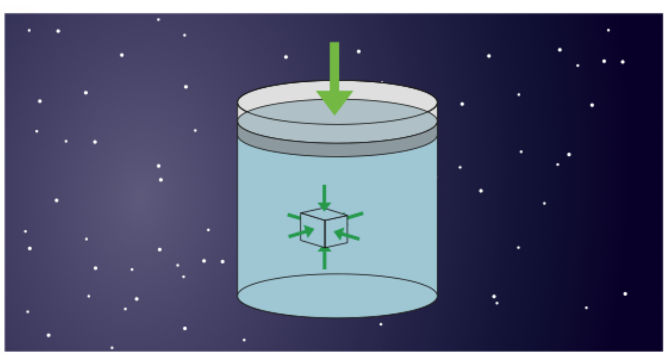
If the container is flexible, like a water balloon, this multi-directional pressure will be evident in the way that the container changes shape when a force is applied.



In the diagram below (there is no gravity!), you can also imagine that the force on the piston is forcing water out of the holding tank in all directions. This only happens if there is pressure in all directions.



Consider the cylinder of liquid shown below in a gravity-free environment. If you push on the piston, it doesn't move. This must be because the force is balanced by the liquid pushing back, right? Now consider the layer of water just below the piston. If the only force acting on it were the applied force from the piston, it would move down. It doesn't, of course, and so that must mean that the next layer of water lower also exerts an upward reaction force.



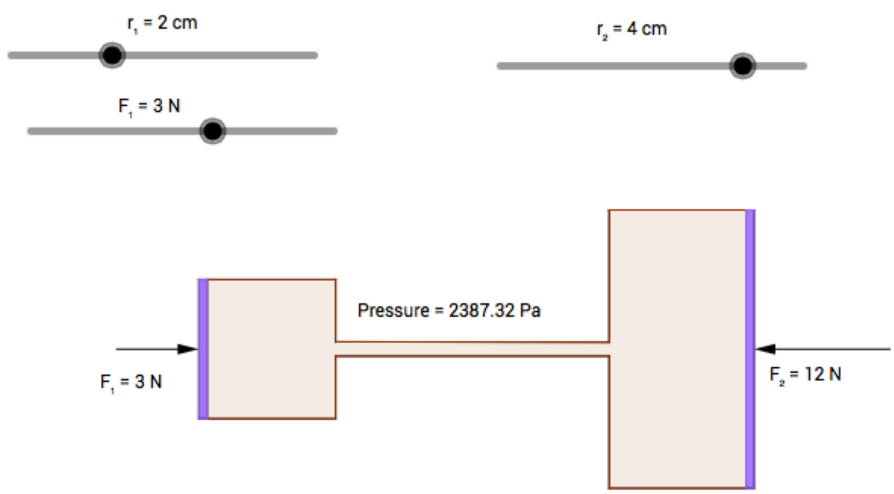
And so on… this continues all the way to the bottom layer of water in the container. The pressure at any point within the liquid is the same in all directions and therefore the fluid doesn't flow.

If two cylinders with different diameters are connected, as below, the pressure everywhere is the same, but because of the difference in area, the resulting force is larger on the larger piston.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Try this out using the GeoGebra simulation "Pascal's Principle".



Exercise

40 A force of 10 N is applied to the small piston of Figure 10.66. If the area of the piston is 1 cm^2 and the area of the big piston is 150 cm^2 calculate:

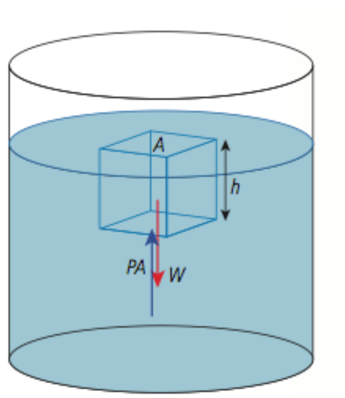
(a) the pressure in the fluid.
(b) the force on the big piston.

Answers: a) 10^5 Pa b) 1500 N

(NOTE: The solutions to all of these questions can be found in the folder I provided you.)

Effect of Gravity

Consider again a cylinder of liquid, this time in a gravitational field, and consider a "cube" of the liquid at the top as shown in the diagram.



Again, the pressure on the cube from all directions is equal except for the bit of liquid shown, there is no pressure on the top of it (assuming no air). Without gravity the liquid would not be held in the container. With gravity, the downward force of gravity balances this. This is called **hydrostatic equilibrium**.

$$\begin{aligned} PA &= mg \\ m &= V\rho = \rho A h \\ PA &= \rho A h g \\ P &= \rho g h \end{aligned}$$

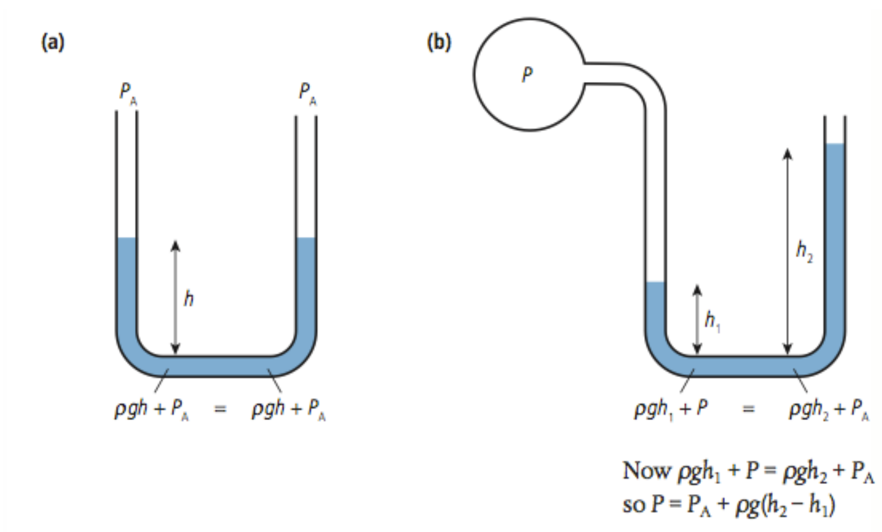
Effect of Atmosphere

Of course, in reality, we also have air on top of the liquid! Air is also a fluid, and exerts pressure on the top of the liquid. According to Pascal's Principle, this will increase the pressure throughout the liquid, so if atmospheric pressure is PA, the pressure at depth h will be:

$$\boxed{P = P_A + \rho g h}$$

The U-tube manometer (The ORIGINAL u tube!)

This device was commonly used to measure air pressure in the days when mercury was in regular use! It consists of a transparent tube containing a liquid.



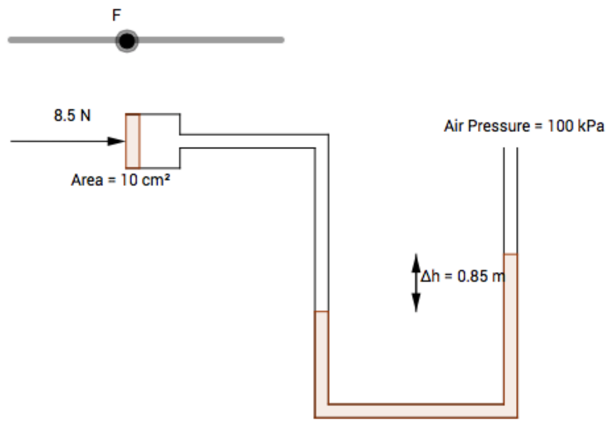
If both ends of the u-tube are open to the atmosphere, we have the situation in diagram (a). The liquid is in equilibrium, and the pressure at the bottom of each column must be equal.

If the pressure on one side of the tube is higher, the height of the liquid on the other side will rise to compensate as in diagram (b). To achieve hydrostatic equilibrium the pressure on both sides must be the same. The pressure on the left is the pressure of the gas and the pressure on the right is the air pressure + the pressure due to the extra height of liquid, $\rho g \Delta h$.

$$\boxed{\Delta P = \rho g \Delta h}$$

If the liquid is water, then when the force exerted on the piston is 10 N, the difference in height will be 1 m. Can you confirm this calculation?

Try it out using the GeoGebra simulation U-Tube Manometer.



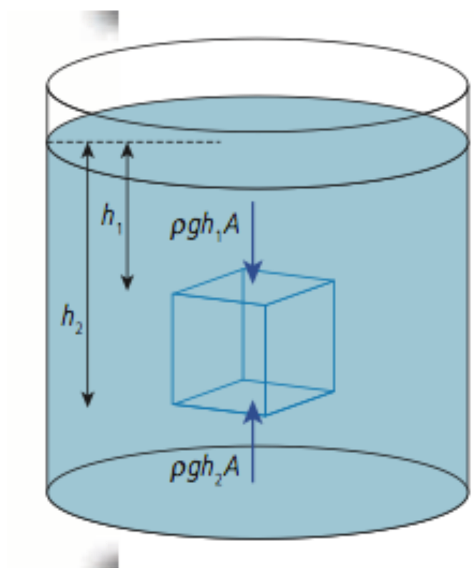
Buoyancy and Archimedes' Principle

Consider once again a cube of liquid like the one shown below:

The bottom surface is deeper than the top, so the pressure will be greater (due to gravity).

If the top depth is h_1 and the bottom depth is h_2, then the difference in force will be:

$$F_2 - F_1 = \rho g h_2 A - \rho g h_1 A = \rho g A(h_2 - h_1)$$



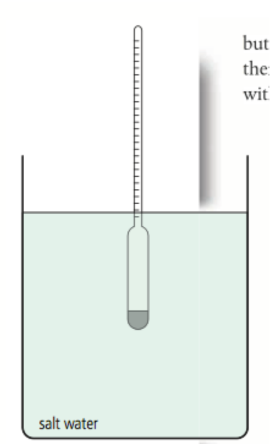
The volume of the cube is given by: $V = A(h_2 - h_1)$

Therefore : $\rho g A(h_2 - h_1)$ is equivalent to the weight of the cube.

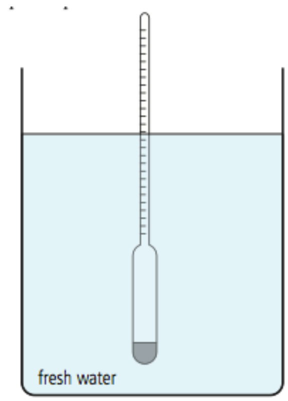
$$\boxed{B = \rho_f V_f g}$$

So long story short….

Archimedes' Principle: The buoyant force on a body immersed in a fluid is equal to the weight of the fluid displaced.



In the two diagrams, the volume of liquid displaced is the same. BUT the weight of liquid displaced on the left is greater (because salt water is more dense) therefore the buoyant force is also greater. This is why it easier for us to float in salt water.



Worked example

The density of water is $1.0 \times 10^3\,\text{kg m}^{-3}$ and the density of iron is $7.8 \times 10^3\,\text{kg m}^{-3}$. Calculate the force required to lift a 60 kg ball of iron from the bottom of a swimming pool.

Solution

First let's draw a diagram of the forces:

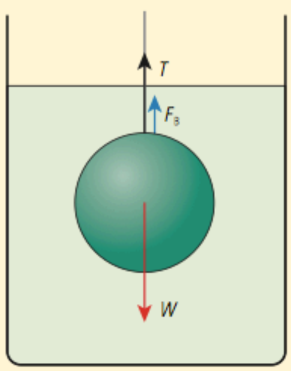


Figure 10.71.

Assuming the ball is lifted at constant velocity the forces are balanced:

$$T + F_B = W = 60 \times 9.8 = 588\,\text{N}$$

F_B = weight of fluid displaced = volume of ball × density of water × g

$$\text{volume of ball} = \frac{\text{mass}}{\text{density of iron}} = \frac{60}{7.8 \times 10^3} = 7.7 \times 10^{-3}\,\text{m}^3$$

$$F_B = 7.7 \times 10^{-3} \times 1.0 \times 10^3 \times 9.8 = 75.4\,\text{N}$$

$$T = W - F_B = 588 - 75.4 = 512.6\,\text{N}$$

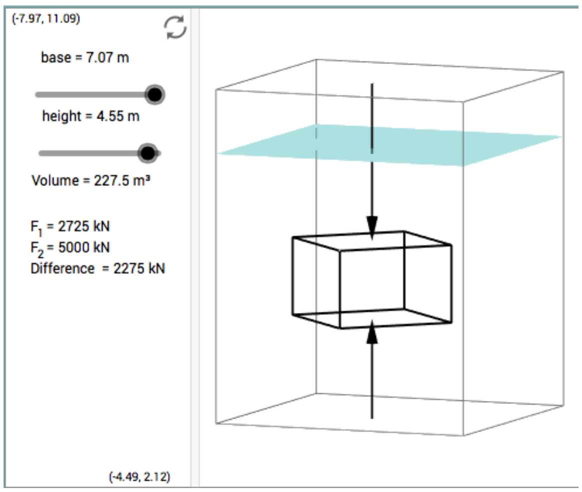
Now you try:

A block is submerged in water as shown in the simulation below. The bottom surface of the block is at a depth of 10 m. The base of the block is square and has side 2 m and the height is 4 m and there is no atmosphere. Calculate:

- The pressure at the level of the bottom of the block.
- The force exerted on the bottom of the block.
- The pressure on the upper surface.
- The force on the upper surface.
- The difference between these forces.

Show that this is the same as the weight of fluid displaced.

To check your answers, use the GeoGebra simulation "Archimedes' principle".



Exercises

41 Calculate the force required to lift a 60 kg ball of gold (density 19.3×10^3 kg m^{-3}) from the bottom of a swimming pool.

42 A 1 m^3 wooden cube floats in water so that 40 cm of the cube is above the water. Calculate:

(a) the density of the wood.
(b) how much force would be required to sink the cube.

43 The density of sea water is 1.03×10^3 kg m^{-3} and the density of ice is 0.92×10^3 kg m^{-3}. Show that when ice floats in sea water 89% is under the surface.

44 A cylinder of gas with a frictionless piston has a volume of 100 cm^3 on the surface of a swimming pool. Calculate its volume at a depth of 5 m.

(atmospheric pressure = 101 kPa)

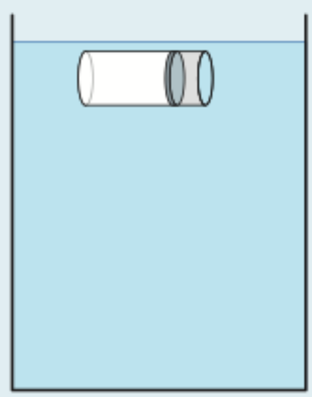


Figure 10.72.

Answers:

41. 559 N 42. a) 600 kgm^{-3} b) 4000 N 44. 67 cm^3

Fluid Dynamics

We will now consider flowing fluids. For simplicity (you know you love it Erick!) we'll look at the simplified case of what's called the **steady flow** of **ideal fluids**.

Let's take a break for a You Tube video, shall we? On the website, entitled "A Non-ideal Fluid".

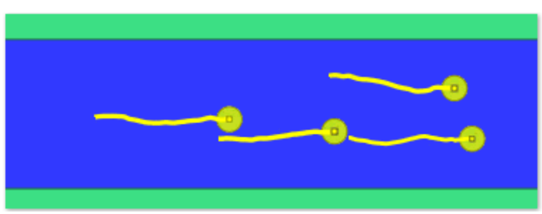
The water in this clip is certainly not flowing steadily, you can see that the water close to the banks is travelling slower than in the middle and there is a lot of turbulence in the fast moving sections (rapids).

It is also not an **ideal fluid**. There is friction between different parts of the liquid. The water in contact with the ground is stationary. This stationary layer slows down the next layer, and so on. This internal friction is called ***viscosity***. It's inconvenient and complicated and annoying … so let's ignore it ☺.

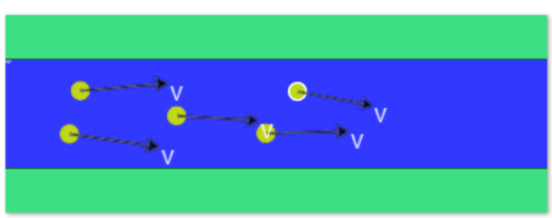
An ideal fluid is incompressible and has zero viscosity.

Streamlines and Flowlines

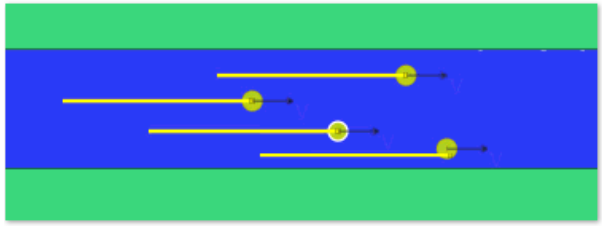
Flowlines – lines that show the path of individual particles of a fluid.



Streamlines – lines that show the velocity of individual particles at any moment in time. In non-steady flow, these are constantly changing.

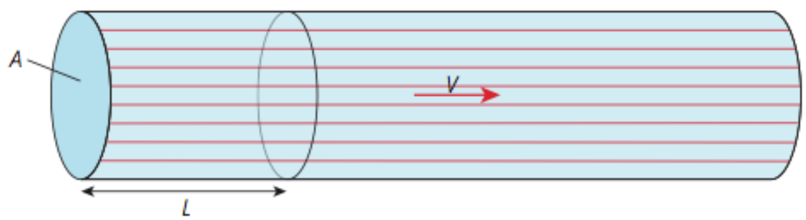


In **steady flow**, the streamlines remain constant, and therefore the flowlines are straight.



The Continuity Equation

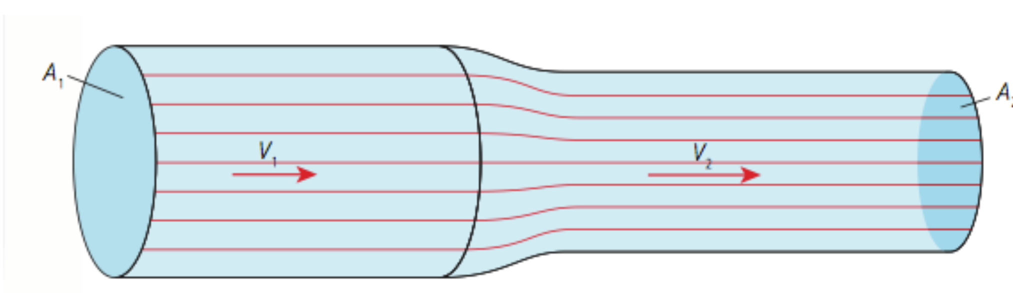
OK, so we have our nice lovely ideal fluid behaving properly and flowing steadily through a nice straight wide pipe ☺. Isn't life lovely.



The cross-sectional area is constant, *A*. The volume that will pass through a given length *L* in a certain time period will be *AL*. So the mass flowing per unit time will be:

$$m = \frac{\rho AL}{\Delta t} = \rho Av$$

If the fluid were to flow into a smaller diameter pipe, the flowlines would get closer together.



Mass flowing in = $\rho_1 A_1 v_1$

Mass flowing out = $\rho_2 A_2 v_2$

If there are no leaks: $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

It's an ideal fluid so it can't be compressed (the density can't change), so:

$$\boxed{A_1 v_1 = A_2 v_2}$$

This is called the ***continuity equation***. The thinner the pipe gets, the faster the flow. (Think about what happens if you squeeze the end of a hose pipe . You probably already know that this will speed up the flow of water – now you know why!)

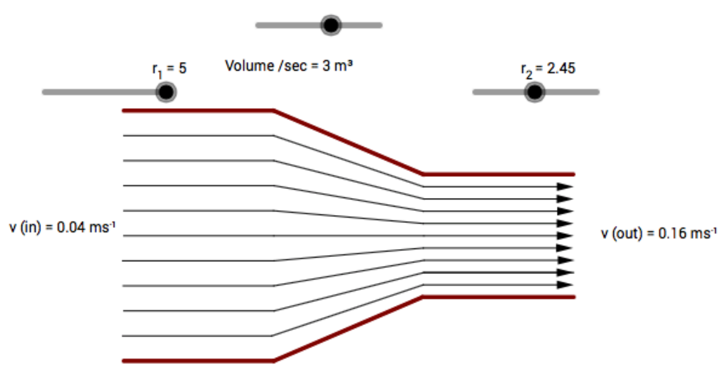
Note: This formula is written on your formula sheet as $Av =$ **constant.**

Example

Fluid flows through a pipe like the one in the diagram

- $3m^3$ of fluid flow into the pipe per second. If the radius of the first pipe is 4 m
  - a) calculate the velocity of the fluid flowing in.
  - b) calculate the velocity of the fluid flowing out if the second pipe has radius 1m.

Check your answers using the GeoGebra simulation "Continuity Equation".



Exercises

45 A river of width 20 m and depth 3 m flows at a speed of 1 $m s^{-1}$. Calculate the increase in speed if the depth changes to 1 m.

46 Water flows through a pipe of diameter 1 m at a rate of 1.5 $m^3 s^{-1}$. The pipe is connected to a second pipe a diameter of 0.5 m. Calculate the speed in the second pipe.

47 A simple water pistol is made out of a cylinder with a moveable piston as shown in Figure 10.76. The cylinder is filled with water and the piston is depressed at a constant speed in 4 seconds. Calculate the speed at which the water is squirted out.

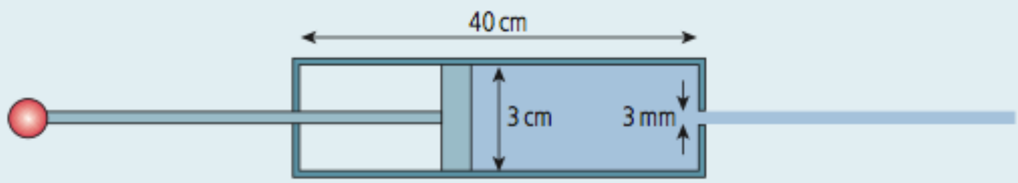


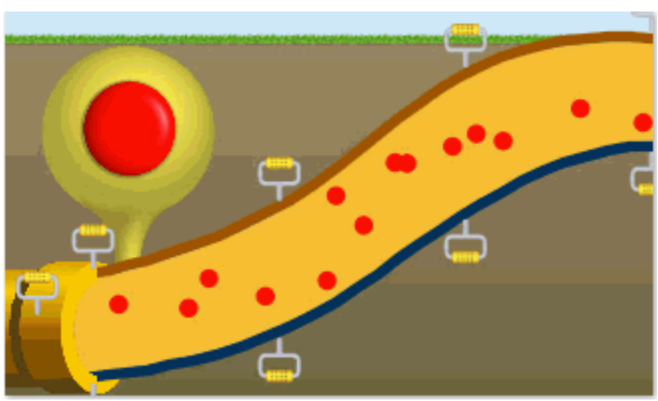
Figure 10.76.

Answers:

45. 3 m/s   46. 7.6 m/s   47. 10 m/s

The Bernoulli Equation

Watch the video entitled "Flow video 1" .



The red dots represent particles of the fluid but they are not *all* shown – the pipe is full.

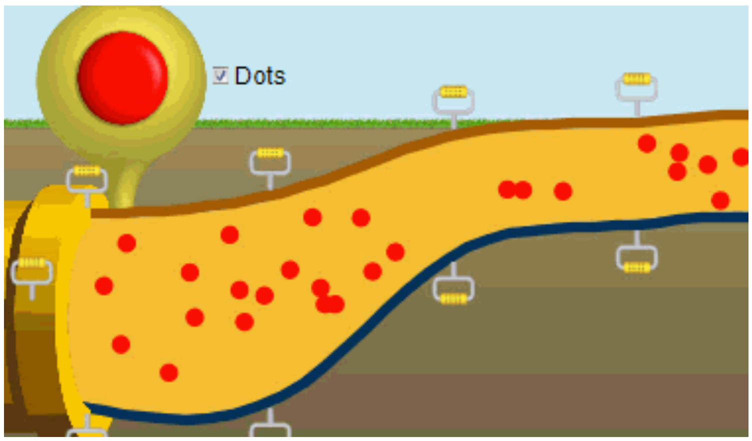
Does the velocity change? (Think about the continuity equation).

What happens to the potential energy of the particles as they go uphill?

Remember, if something gains energy, **work** must be done.

(You can download this simulation for yourself here:
https://phet.colorado.edu/en/simulation/fluid-pressure-and-flow
or play with it on my website. It's fun to play around with!)

Now watch this second video, "Flow video 2".



The velocity of the particles clearly increases. Why?

What energy changes take place?

Is work done on the water?

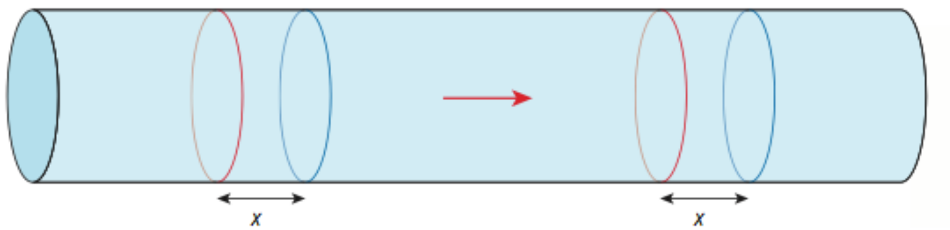
Both KE and PE increase, so clearly work must be done on the water.

Something must be pushing the fluid, like a pump or a piston.

We know that when a fluid flows into a section of pipe with smaller cross-sectional area its velocity increases. This means acceleration occurs, which means there is an unbalanced force. Therefore the pressure on the slow-moving side must be less than the pressure on the fast moving side.

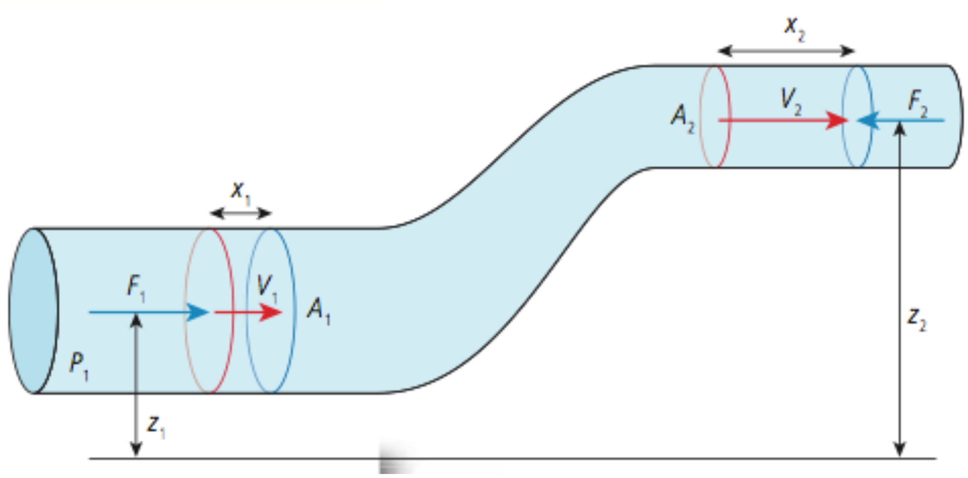
We also know that if a fluid flows in a vertical pipe, the pressure is greater at the bottom than at the top.

Let's look first at a uniform horizontal pipe:



A cross-section of water progresses a distance x in some period of time. This distance is the same at both the left hand side or right hand side of the pipe.

Consider now the following diagram:



In this case, water is flowing uphill in a narrowing pipe. A volume of fluid on the right hand side will progress a greater distance than the same volume on the left hand side because it moves faster in the narrower part.

Also, as the fluid rises it gains both PE and KE so there is work being done on it.

Water from beyond the edges of pipe shown exerts pressure at each end, but F_1 (on the diagram) is larger than F_2 or the fluid wouldn't be flowing.

Net work done = Work done on the water at the bottom – work done on the water at the top

$$W = F_1x_1 - F_2x_2 = P_1A_1x_1 - P_2A_2x_2$$

Because Ax = V = mass/density:

$$W = \frac{P_1 m}{\rho} - \frac{P_2 m}{\rho}$$

The work done is the change in energy therefore:

$$W = \Delta PE + \Delta KE$$

We equate the 2 formulas to get:

$$\frac{P_1 m}{\rho} - \frac{P_2 m}{\rho} = mgz_2 - mgz_1 + \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

where z is the height of the fluid.

which rearranges to give us:

$$\boxed{P_1 + \tfrac{1}{2}\rho v_1^2 + pgz_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + pgz_2}$$

This is **the Bernoulli equation** and can be applied along any streamline.

Worked example

Water flows into the bottom of a section of pipe similar to Figure 10.78 with a pressure of 4×10^5 Pa and speed 2.0 m s^{-1}. Calculate the speed and pressure of the water at the top if the diameter of the bottom is 2.0 cm, the diameter of the top 1.0 cm, and the height difference 5 m.

Solution

To calculate the velocity at the top we can use the continuity equation $A_1v_1 = A_2v_2$
Assuming the pipes are circular cross section

$$\pi \times (1 \times 10^{-2})^2 \times 2 = \pi \times (0.5 \times 10^{-2})^2 \times v_2$$
$$v_2 = 8 \text{ m s}^{-1}$$

Now we can use the Bernoulli equation to find the pressure

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g z_2$$

If we take the lower pipe at ground level then $z_1 = 0$

$$4 \times 10^5 + \tfrac{1}{2} \times 1000 \times 2^2 + 0 = P_2 + \tfrac{1}{2} \times 1000 \times 8^2 + 1000 \times 10 \times 5$$

$$P_2 = 400\,000 + 2000 - 32\,000 - 50\,000 = 320\,000 \text{ Pa} = 3.2 \times 10^5 \text{ Pa}$$

Your turn:

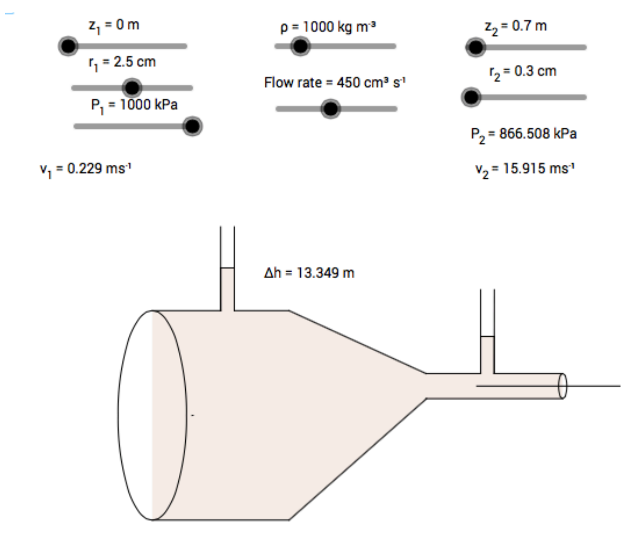
Water flows into a pipe of radius 2 cm at a rate of 100 cm^3s^{-1} at a pressure of 500 kPa.

- Calculate the velocity of the water.

The pipe is connected to a thicker pipe of radius 5 cm that is 4m above the height of the first one.

- Calculate the velocity in the second pipe.
- Calculate the pressure of the water in the top pipe.

Use the GeoGebra simulation "Bernoulli equation" (note that the radius and height are not the same scale) to check your answers.



Exercises

48 Water with speed 3 m s^{-1} at a pressure of 500 kPa flows through a horizontal pipe that widens from diameter 2 cm to 6 cm. Assuming the fluid to be ideal, calculate:

(a) the volume flowing per second.
(b) the velocity of the water in the wider pipe.
(c) the pressure in the wider pipe.

49 Water flows from a water tank at the top of a building to a washroom 20 m below. The water enters a pipe with diameter 3 cm at a pressure 100 kPa travelling at 0.5 m s^{-1}. The last metre of pipe connected to the tap in the washroom has a diameter of 1 cm. Calculate:

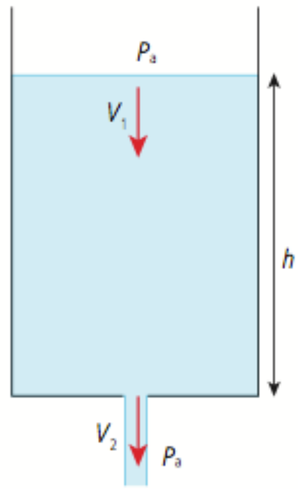
(a) the volume flowing per second.
(b) the velocity of water through the pipe connected to the tap.
(c) the pressure in the pipe connected to the tap.

Answers:

48. a) 9.4×10^{-4} m^3/s b) 0.33 m/s c) 504.4 kPa
49. a) 3.53×10^{-4} m^3/s b) 4.5 m/s c) 290 kPa

Hole in a Bucket (Torricelli's Theorem)

Consider a container with a hole in the bottom:



Because the top and bottom are both open, the pressure is the same as atmospheric pressure at both ends.

$$P_a + \tfrac{1}{2}\rho v_1^2 + pgh = P_a + \tfrac{1}{2}\rho v_2^2 + pg \times 0$$

P_a cancels on each side, and if the bucket is big and the hole is small, we can approximate that $v_1 = 0$. (You know you love it, Erick)

So we end up with:

$$v_2 = \sqrt{2gh}$$

The Venturi Meter

The pressure difference when a fluid flows through a constriction can be used to measure fluid flow.

Have a look again at the GeoGebra simulation for the Bernoulli equation. You'll notice the vertical tubes (manometers). They can be used to measure the pressure in the pipe.

- Set both pipes at the same height.
- Set the first pipe to 1.6 cm and the second to 0.4 cm

- Set the flow rate to 800 cm3s-1 the density to 1000 kgm-3 and P1 to 1000 kPa
- Note the difference in height of the manometers.
- Reduce the flow rate to 400 cm3s-1 and observe the change in Δh.

This arrangement can be used to measure the flow of liquid in a pipe.

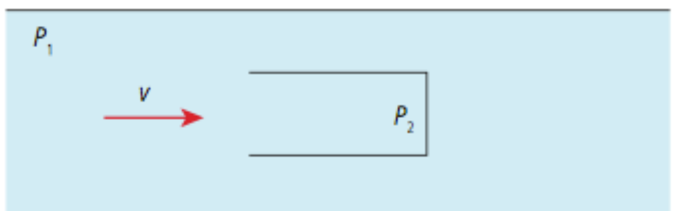
Show that the equation

$$g\Delta h = \frac{1}{2}v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Agrees with the simulation.

Stagnation Pressure

According to the Bernoulli equation, when a fluid is brought to rest the pressure must increase.



Any fluid that flows into the stationary closed pipe will stop, therefore:

$$P_1 + \frac{1}{2}\rho v^2 = P_2$$

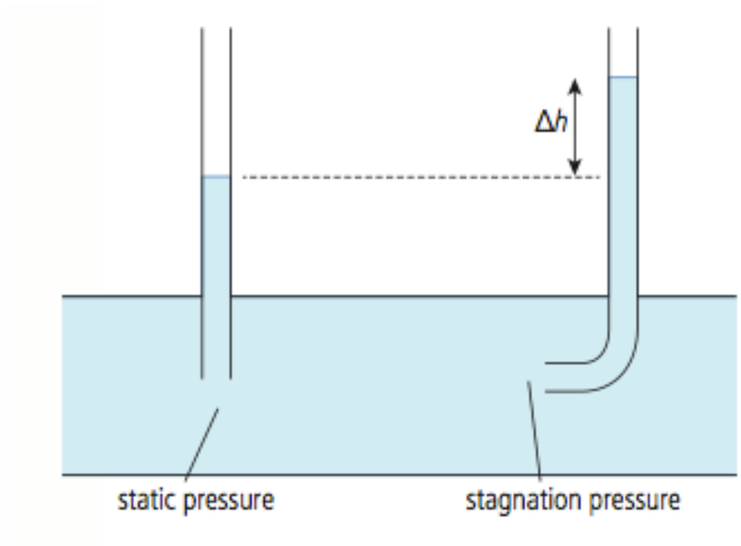
So we can see that the pressure in the closed pipe (P_2) is greater than the pressure elsewhere (P_1). This pressure is called ***the stagnation pressure.***

If we measure the pressure difference, it will tell us the flow rate:

$$P_2 - P_1 = \frac{1}{2}\rho v^2$$

Pitot Static Tube

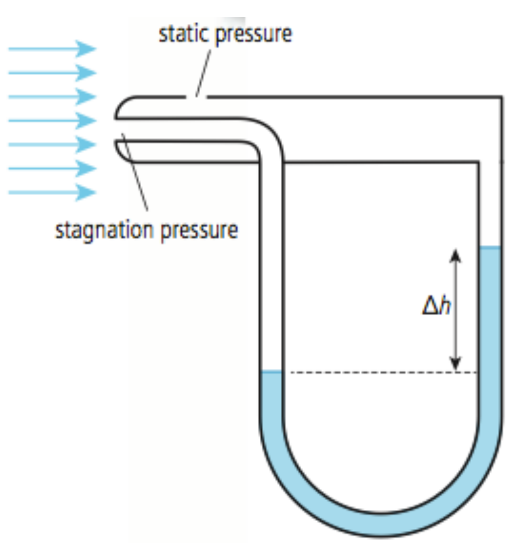
This device measures the difference in pressure of a flowing fluid and stagnation pressure in order to determine flow rate.



Apply Bernoulli's equation to the streamline from the end of the left pipe to the bend in the right pipe and show that:

$$P_2 - P_1 = \frac{1}{2}\rho v^2 = \rho g \Delta h$$

A similar design can be used to measure the speed of gases, like shown below. This device can measure the speed of an airplane.



Exercises

50 A pitot tube as in Figure 10.83 with a water-filled manometer is used to measure the wind speed in an air tunnel. Calculate the speed if the difference in height of the manometer columns is 3 cm.

density of air = 1.3 kg m^{-3}
density of water = 1000 kg m^{-3}

51 Calculate the difference in pressure in a pitot tube used to measure the speed of an airplane travelling at 600 km h^{-1}.

52 Calculate the volume of water flowing through the Venturi meter shown in Figure 10.84.

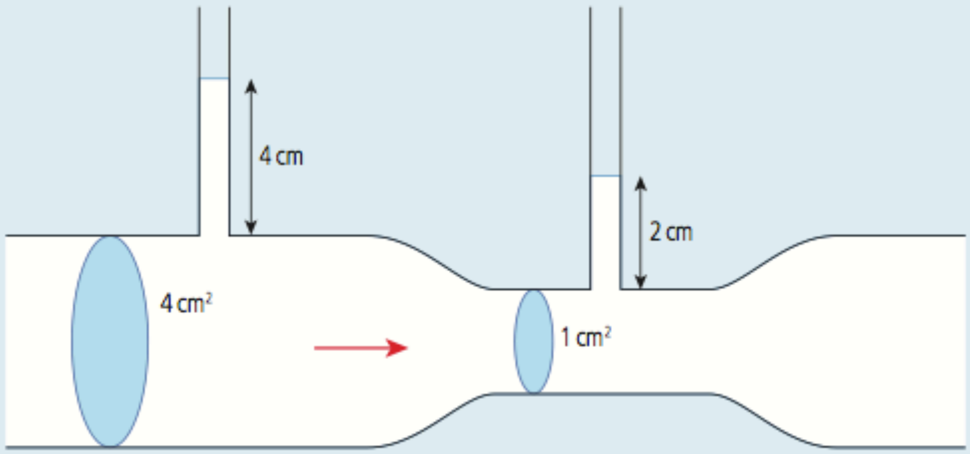


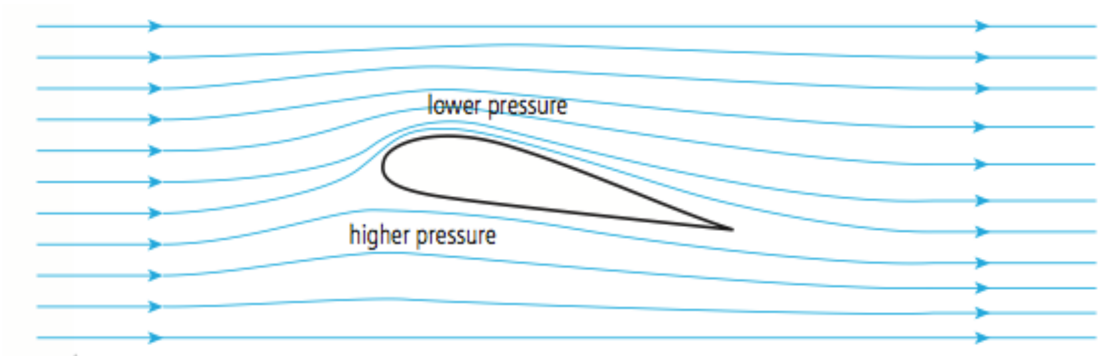
Figure 10.84 A Venturi meter.

Answers:

50. 21.5 m/s 51. 18 kPa 52. 6.5 x 10^{-5} m^3/s

The Bernoulli Effect

The most obvious example of the Bernoulli effect is the lift-off of an airplane. Because of the shape of the wing, air travelling over the top of the wing has a higher velocity than underneath it. The faster moving air, according to Bernoulli, has lower pressure, so an upward force results.



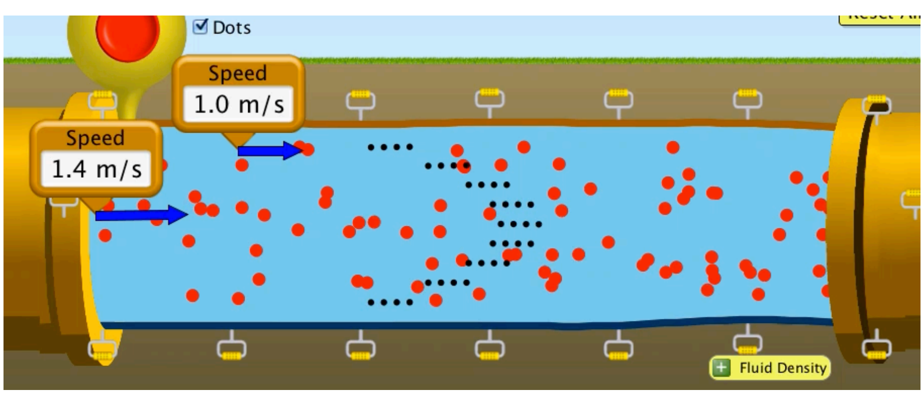
(This is a simplified, incomplete story of how a plane gets off the ground, by the way).

There are other cool examples of the Bernoulli effect, such as bending it like Beckham. Maybe you can put your mad You Tube skills into action and learn a bit more.

Real Fluids

OK, OK, so fluids don't REALLY have zero viscosity and flow steadily like we might like them too! In reality they have internal friction and therefore viscosity.

Watch "Flow video 3" and notice the difference in the flow rate near the edges of the pipe compared to the centre. (I did this by turning friction on, by the way.)



Flow like that shown in the video is called **lamina flow** – the layers do not mix. This only happens at low velocities.

Viscosity

Viscosity is defined in terms of the force between two plates moving parallel to each other separated by the fluid.

The force require to move the plate is proportional to its velocity (v) the area of the plate (A) and the separation of the plates L, so:

$$F \propto \frac{Av}{L}$$

Which gives:

$$\eta = \frac{FL}{vA}$$

The constant of proportionality η is the viscosity. What are the correct units?

Stoke's Law

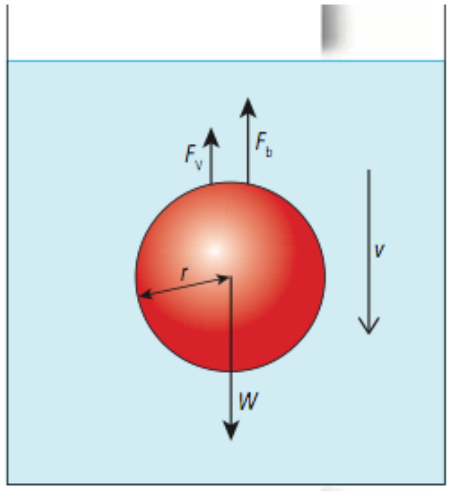
When bodies move through fluids, they experience an opposing force due to the viscosity of the fluid. The magnitude of such a force on a sphere is proportional to the size of the sphere and its speed.

$$\boxed{F = 6\pi\eta rv}$$

Terminal Velocity

An object falling due gravity is falling through a fluid (air). According to Stoke's Law, there will be a force opposing its motion (air resistance as we called it earlier, but also known as the ***viscous force***) proportional to its velocity. As we know, that force will increase until such time that it equals the force due to gravity resulting in zero net force, and the object stops accelerating – it reaches terminal velocity.

The same concept can be applied to an object falling in a different fluid, say water. There are two upward forces: buoyancy and viscous force.



Terminal velocity is reached when the sum of the buoyant and viscous forces is equal to the weight of the object.

$$F_v + F_b = F_g$$

From Stoke's Law:

$$F_v = 6\pi\eta r v_t$$

And if we replace F_b by the weight of fluid replaced (recall Archimedes' principle) and F_g by the weight of the object:

$$6\pi\eta r v_t + \tfrac{4}{3}\pi r^3 \rho_f g = \tfrac{4}{3}\pi r^3 \rho_s g$$

And solving for terminal velocity gives us:

$$v_t = \frac{2}{9}\frac{gr^2}{\eta}(\rho_s - \rho_f)$$

Turbulent Flow

When the velocity reaches a certain point, the flow will no longer remain laminar, but will become turbulent (the layers will mix). This will occur when the **Reynolds number** (R_e) exceeds 1000.

$$\boxed{R_e = \frac{vr\rho}{\eta}}$$

What would the units be for the Reynolds number?

Exercises

53 Calculate the terminal velocity of a 0.5 cm radius steel ball falling through oil.
density of oil = 900 kg m^{-3}
density of steel = 8000 kg m^{-3}
viscosity of oil = 0.2 N s m^{-2}

54 Calculate the density of a ball of radius 3 cm that has a terminal velocity of 0.5 m s^{-1} falling through the same oil as in Exercise 53.

55 Calculate the volume flow rate at which the flow of water through a 2 cm diameter pipe becomes turbulent.

viscosity of water = 0.002 N s m^{-2}

Answers:
53. 1.97 m/s   54. 950 kg/m^3 55. 6.3 x 10^{-5} m^3/s